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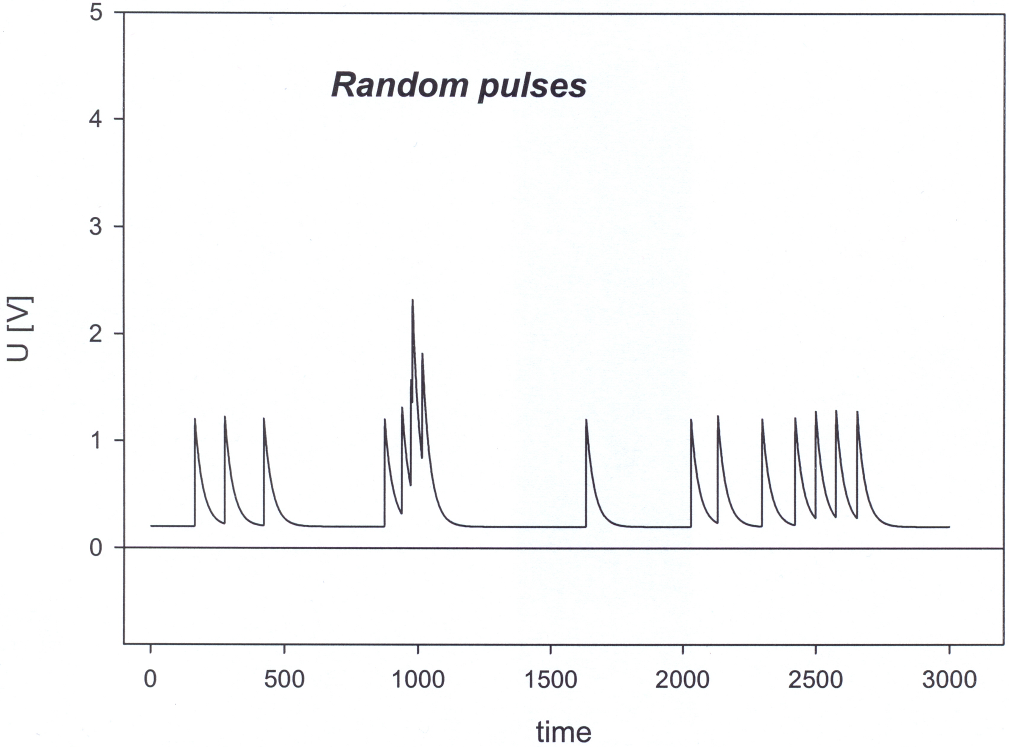
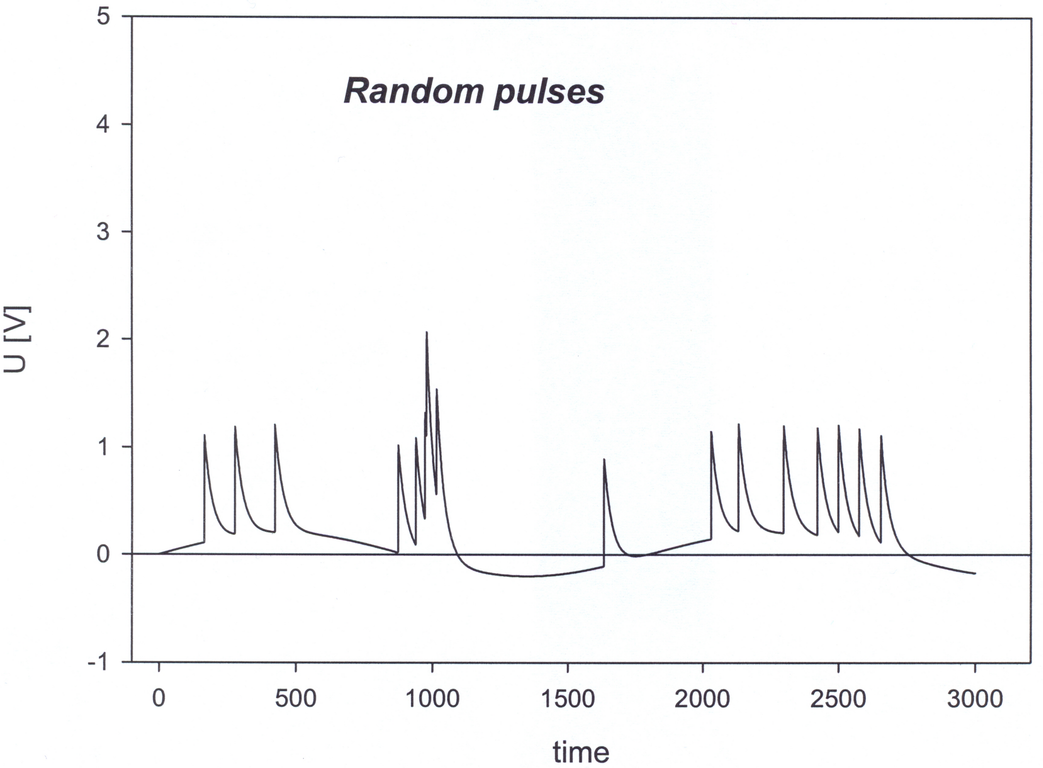
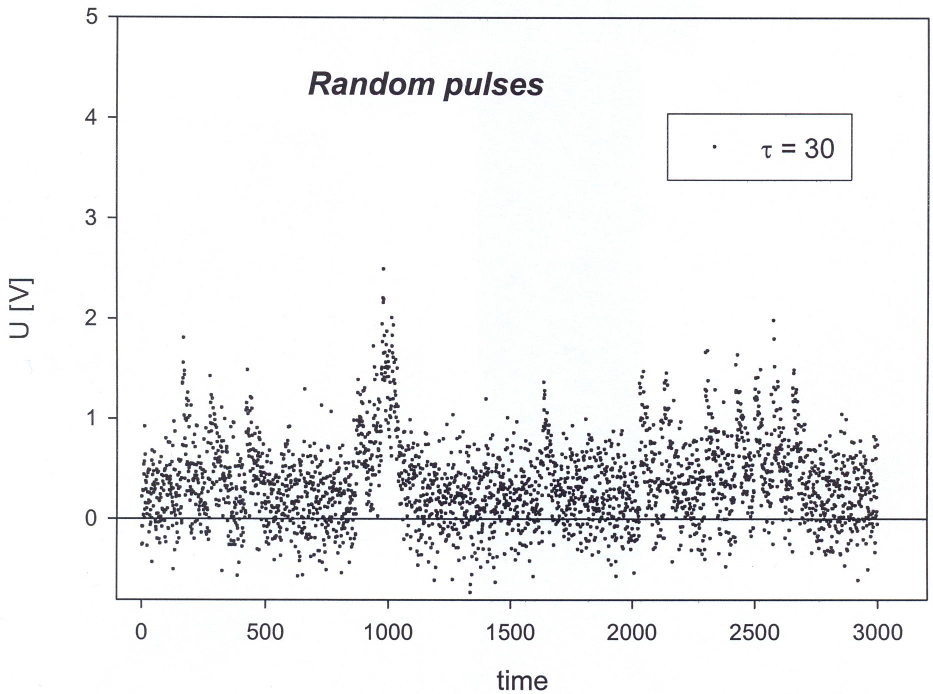
# An optimal filter for exponential pulses of very high rates

FREEDAC Meeting, Ljubljana, May 28$^{th}$-30$^{th}$, 2008


Random pulses
U [V]
5
4
3
2
1
0
0
500
1000
1500
2000
2500
3000
time



Random pulses
U [V]
5
4
3
2
1
0
-1
0
500
1000
1500
2000
2500
3000
time


Random pulses
τ = 30
U [V]
time
0
1
2
3
4
5
0
500
1000
1500
2000
2500
3000

# Fundamentals

$$\frac{dP}{dz} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(z-x)^2}{2\sigma^2}}$$

$$\overline{z_1} = \frac{1}{n}\sum_{i=1}^{n} z_i \tag{1}$$

$$\sigma_1^2 = \frac{\sigma^2}{n} \tag{2}$$

$$\overline{z_2} = \frac{1}{m} \sum_{i=n+1}^{n+m} z_i, \qquad \sigma_2^2 = \frac{\sigma^2}{m} \tag{3}$$

$$\overline{x} = \frac{1}{n+m} \sum_{i=1}^{n+m} z_i, \qquad \sigma_{\bar{x}}^2 = \frac{\sigma^2}{n+m} \tag{4}$$

$$\overline{x} = \frac{1}{n+m} \left( \sum_{i=1}^{n} z_i + \sum_{i=n+1}^{n+m} z_i \right) ,$$

$$\overline{x} = \frac{n}{n+m}\overline{z_1} + \frac{m}{n+m}\overline{z_2}$$

$$\sigma_{\bar{x}}^{-2} = \frac{n}{\sigma^2} + \frac{m}{\sigma^2} = \sigma_1^{-2} + \sigma_2^{-2} ,$$

$$n = \frac{\sigma^2}{\sigma_1^2}, \qquad m = \frac{\sigma^2}{\sigma_2^2}\,.$$

$$\left.\begin{aligned} \overline{x} &= \frac{\sigma_2^2}{\sigma_1^2+\sigma_2^2}\overline{z_1} + \frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\overline{z_2}\,, \\ \sigma_{\bar{x}}^{-2} = \sigma_1^{-2} + \sigma_2^{-2}\,, &\qquad \sigma_{\bar{x}}^2 = \frac{\sigma_1^2\sigma_2^2}{\sigma_1^2+\sigma_2^2}\,. \end{aligned}\right\} \tag{5}$$

$$\overline{x} = \overline{z_1} + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}(\overline{z_2} - \overline{z_1}) \tag{6}$$

# Filter with dynamics

$$\dot{x} = A(t)x + c(t)$$

$$\dot{x} \approx \frac{x_{n+1} - x_n}{T}$$

$$x_{n+1} = x_n + A(nT)Tx_n + c(nT)T$$

$$\Phi_n = 1 + A(nT)T\,, \quad c_n = c(nT)T$$

$$x_{n+1} = \Phi_n x_n + c_n \tag{7}$$

# Extrapolation

$$\overline{x}_{n+1} = \Phi_n \widehat{x}_n + c_n \tag{8}$$

$$\overline{\sigma}^2_{n+1} = \langle (\overline{x}_{n+1} - x_{n+1})^2 \rangle = \Phi_n^2 \langle (\widehat{x}_n - x_n)^2 \rangle = \Phi_n^2 \widehat{\sigma}^2_n$$

$$\overline{\sigma}^2_{n+1} = \Phi_n^2 \widehat{\sigma}^2_n \tag{9}$$

$$\widehat{\sigma}^2_n = \langle (\widehat{x}_n - x_n)^2 \rangle$$

New information $z_{n+1}$ from the sensor

$$\widehat{\sigma}_{n+1}^2 = \frac{\overline{\sigma}_{n+1}^2 \sigma^2}{\overline{\sigma}_{n+1}^2 + \sigma^2} \tag{10}$$

$$\widehat{x}_{n+1} = \overline{x}_{n+1} + \frac{\overline{\sigma}_{n+1}^2}{\overline{\sigma}_{n+1}^2 + \sigma^2}(z_{n+1} - \overline{x}_{n+1}) \tag{11}$$

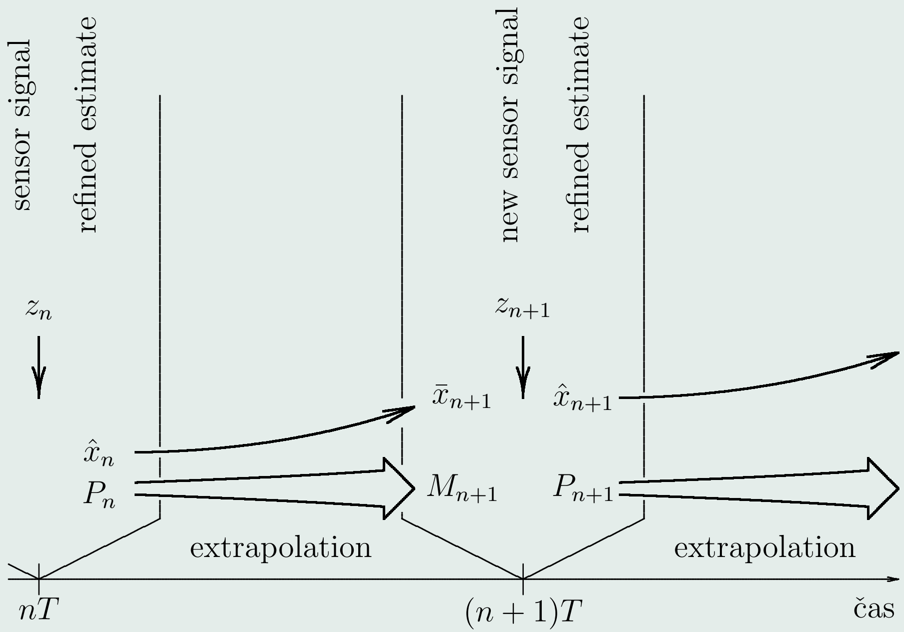
# Optimal (Kalman) filter

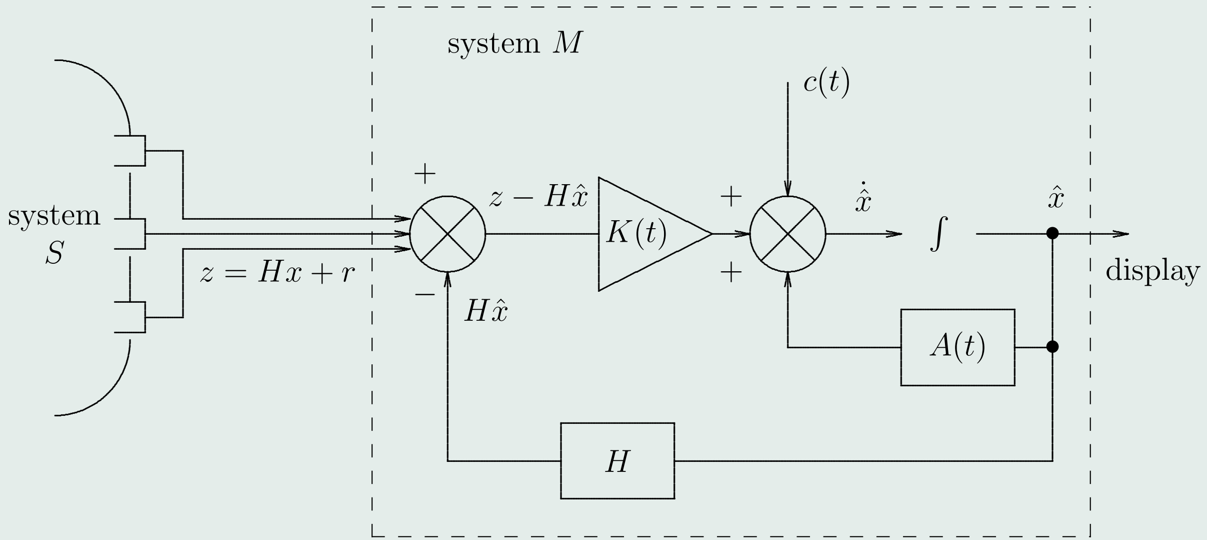
$$\left.\begin{aligned}
M_{n+1} &= \Phi_n^2 P_n \,, \\
P_{n+1} &= \frac{M_{n+1}\sigma^2}{M_{n+1}+\sigma^2} = M_{n+1} - \frac{M_{n+1}^2}{M_{n+1}+\sigma^2} \,, \\
K_{n+1} &= \frac{P_{n+1}}{\sigma^2} \,, \\
\\
\overline{x}_{n+1} &= \Phi_n \widehat{x}_n + c_n \,, \\
\widehat{x}_{n+1} &= \overline{x}_{n+1} + K_{n+1}(z_{n+1} - \overline{x}_{n+1}) \,.
\end{aligned}\right\} \qquad (12)$$

$$c_n \longrightarrow c_n + \mathit{\Gamma}_n w_n$$

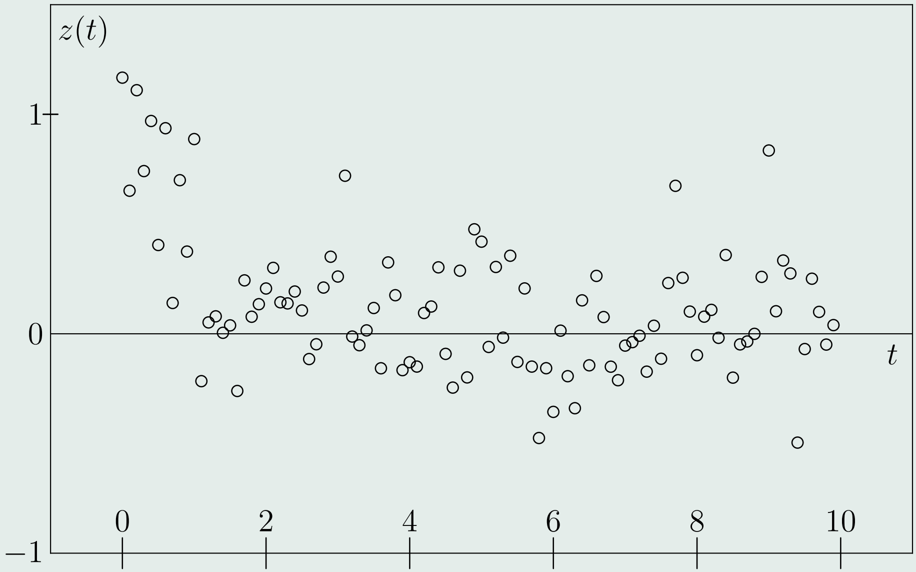
$$\langle w_n w_{n'} \rangle = \delta_{nn'} Q_n$$

$$\begin{aligned} M_{n+1} &= \langle [\mathit{\Phi}_n(\widehat{x}_n - x_n) - \mathit{\Gamma}_n w_n][\mathit{\Phi}_n(\widehat{x}_n - x_n) - \mathit{\Gamma}_n w_n] \rangle \\ M_{n+1} &= \mathit{\Phi}_n^2 P_n + \mathit{\Gamma}_n^2 Q_n \end{aligned}$$

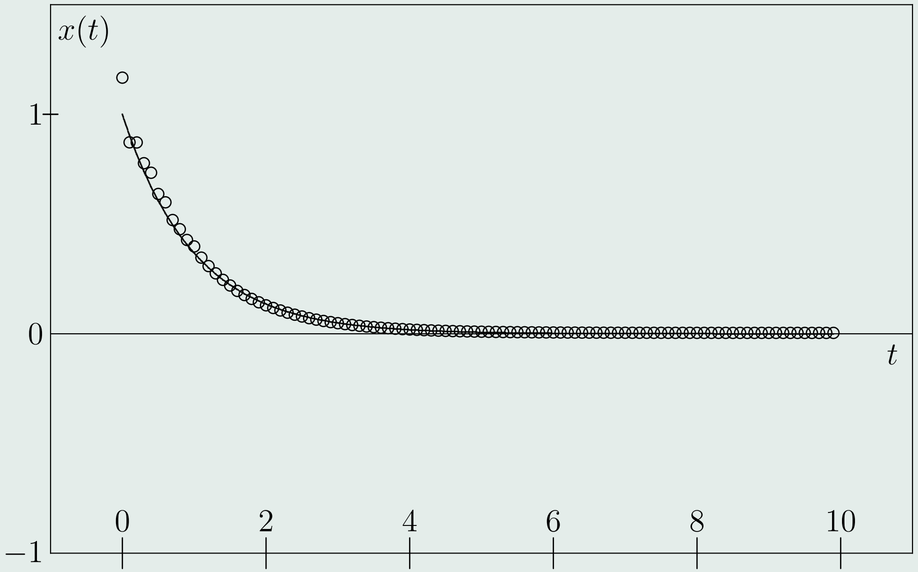

sensor signal
refined estimate
new sensor signal
refined estimate
$z_n$
$z_{n+1}$
$\bar{x}_{n+1}$
$\hat{x}_{n+1}$
$\hat{x}_n$
$P_n$
$M_{n+1}$
$P_{n+1}$
extrapolation
extrapolation
$nT$
$(n+1)T$
čas



system $M$
$c(t)$
system
$S$
$+$
$z - H\hat{x}$
$K(t)$
$+$
$\dot{\hat{x}}$
$\int$
$\hat{x}$
$+$
display
$z = Hx + r$
$-$
$H\hat{x}$
$A(t)$
$H$



$z(t)$
1
0
−1
0
2
4
6
8
10
$t$



x(t)
1
0
−1
t
0
2
4
6
8
10

# The algorithm

$$\mathbf{H}(\mathbf{t}) = [e^{-\frac{t}{\tau}}, 1]$$

$$z = \mathbf{H}(t)\mathbf{x} + r = Ae^{-\frac{t}{\tau}} + b + r$$

$$t = 0$$

at main trigger

**DO WHILE** $1 \leq 2$

test for the second trigger:

if the first one =1 and $|z - \mathbf{H}\overline{\mathbf{x}}| > 0$

test for the first trigger:

if main trigger=0.and. $|z - \mathbf{H}\overline{\mathbf{x}}| > d * \sigma_r$

MAIN TRIGGER:

if the second trigger = 1 .and. the first =1 then

$\mathbf{K}(t = 0)$, store $A_{n-2}$ to the spectrum

refinement for $A_{n-2}$ and $b_{n-2}$

extrapolation of $A_{n-2}$ and $b_{n-2}$ (trivial as $\dot{A} = \dot{b} = 0$)

**END DO**

$\sigma_r = 0.05$

random pulses

gafila05 glej05

$\sigma_r = 0.2$

random pulses

gafila2 glej2

$$\sigma_r = 0.2$$

even pulses

gafila2e glej2e

$$\sigma_r = 0.5$$

random pulses

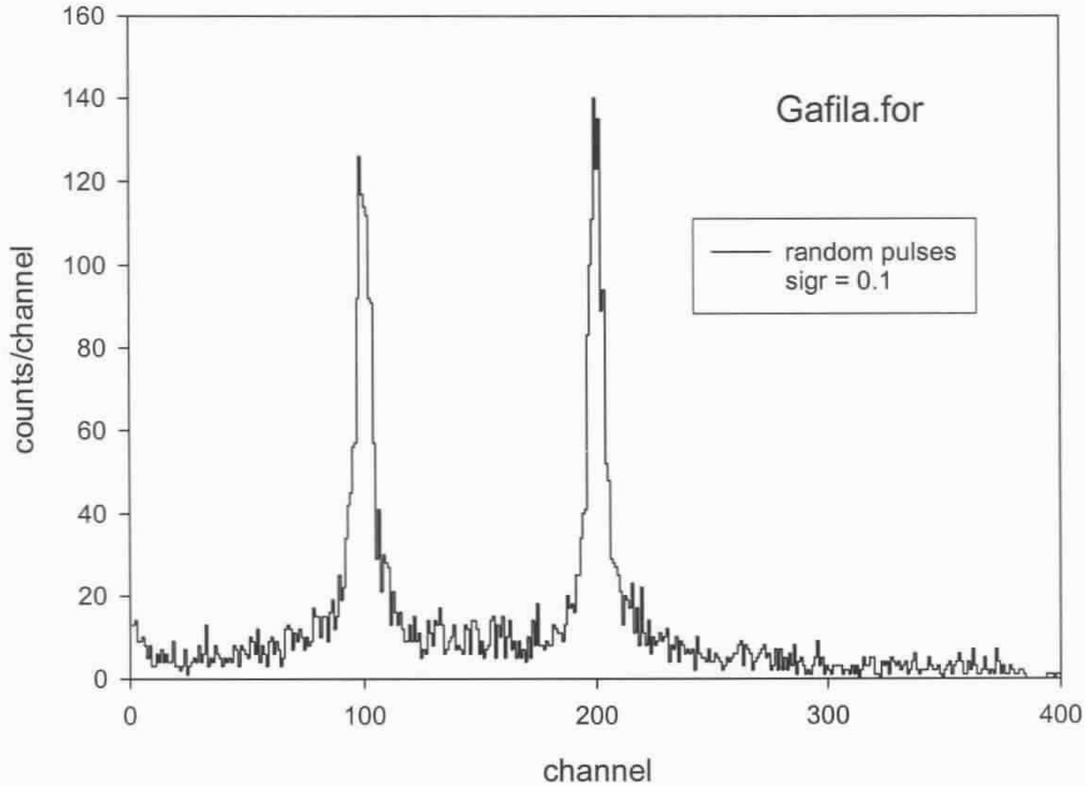
gafila5 glej5

A bimodal Kalman smoother for nuclear spectrometry

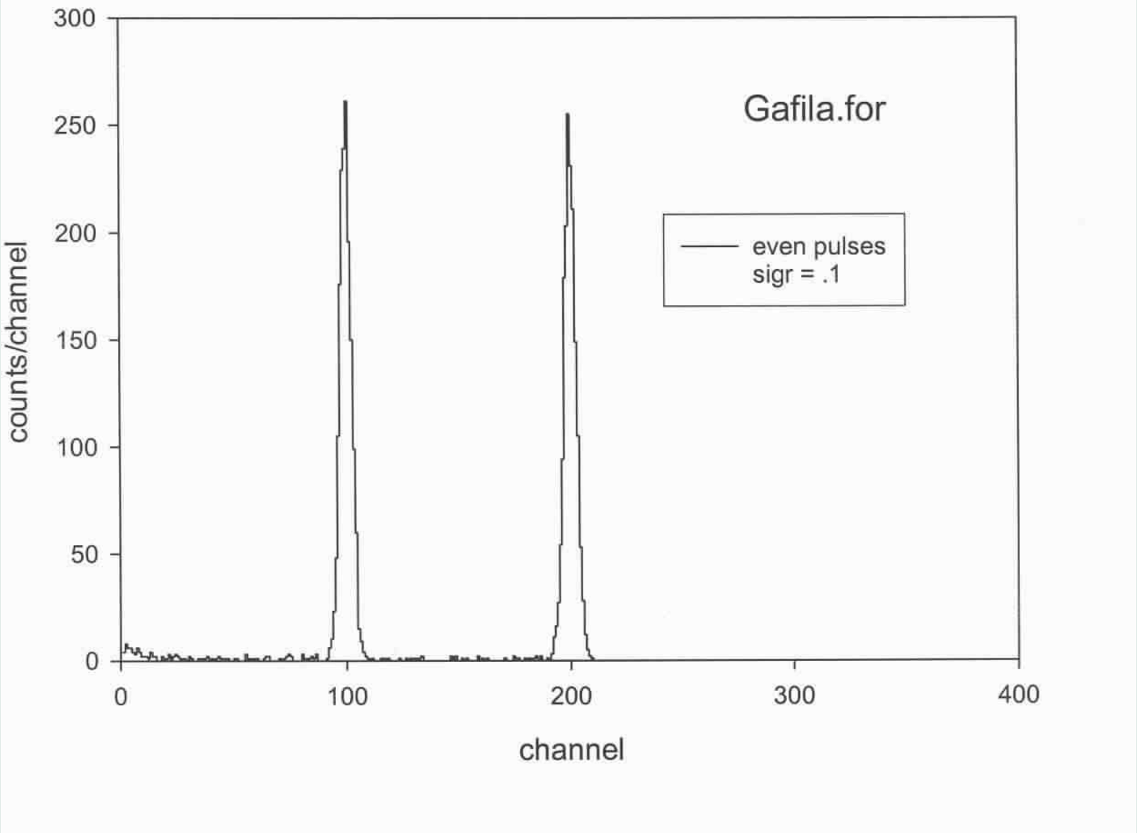
E. Barat, T. Dautremer, T. Montagu, J.-C. Trama, NIM A 567 (2006) 350-352

$$\left.\begin{aligned} p_{k+1} &= r_k w_k^p , \\ b_{k+1} &= b_k + w_k^b , \\ n_{k+1} &= -\alpha w_{k-1}^n + w_k^n , \\ y_{k+1} &= p_k + b_k + n_k . \end{aligned}\right\} \qquad (13)$$

NIM A



Gafila.for
random pulses
sigr = 0.1
counts/channel
channel
0
20
40
60
80
100
120
140
160
0
100
200
300
400



Gafila.for
even pulses
sigr = .1
counts/channel
channel
0
50
100
150
200
250
300
0
100
200
300
400